Interactions between polarized soliton pulses in optical fibers: Exact solutions

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We derive an exact expression for the two-soliton solution to the Manakov equations, including the practically important case of two merging eigenvalues. This solution is a useful tool, e.g., for analysis of interaction between arbitrarily polarized soliton pulses in optical fibers. [S1063-651X(96)13611-4]

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I. INTRODUCTION

Soliton interactions are a class of phenomena that have been studied extensively over the years. There are several reasons for this. First, in the diverse fields in which solitons play a dominant role (including many important applications to nonlinear optics, solid state, plasma physics, fluid dynamics, etc.), the interaction may be a crucial factor in determining physical properties of the soliton-bearing systems [1]. In particular, it is often necessary to have a detailed understanding of the interaction processes in order to draw a full benefit from the solitons in applications. Important examples are the soliton-based optical communication systems [2] and nonlinear optical switches [3]. Second, the completely elastic interactions between solitons in integrable systems are a unique property that distinguishes genuine solitons from ordinary, stable solitary waves.

In this paper we investigate soliton interactions in the Manakov model. The Manakov equations describe, under certain conditions, nonlinear interaction between the orthogonal polarization components of an electromagnetic wave propagating in a nonlinear Kerr medium [4]. They can be viewed as an integrable generalization of the nonlinear Schrödinger (NLS) equation [5]. In optical fibers, the Manakov system arises in certain polarization-preserving birefringent fibers [6], and, more importantly, in long fibers with randomly rotating polarization axes, which is the case for most standard fibers; in this case, the Manakov equations appear as a result of averaging over the rapid random rotation of the axes [7–9]. In a normalized form, the Manakov system can be written as follows:

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial t^2}+(|u|^2+|v|^2)u=0, \tag{1}$$

$$i\frac{\partial v}{\partial z}+\frac{1}{2}\frac{\partial^2 v}{\partial t^2}+(|u|^2+|v|^2)v=0, \tag{2}$$

where u and v are envelopes of the two polarization components of the electromagnetic field. In these equations, we use the standard ''optical'' notation [2], i.e., the evolutional variable z is the propagation distance along the fiber, while t is the so-called retarded time.

Equations (1) and (2) have attracted renewed attention in recent theoretical [9–11] and experimental [10] investigations of interactions between optical solitons in fibers. In this work, we will describe some physically relevant analytical solutions (the two-soliton solution) to Eqs. (1) and (2) by means of the inverse scattering transform (IST). These explicit analytical solutions are likely to be very useful when interaction between arbitrarily polarized soliton pairs in optical fibers is studied. In that sense, this work is a generalization of Gordon's classical work on soliton interaction in the scalar (NLS) approximation [12]. Obviously the complexity of the two-soliton solution is much more involved in the vectorial case, but we will show below that important physical conclusions can be drawn from these solutions anyway.

The IST for Eqs. (1) and (2) was found by Manakov [4], who showed that the associated scattering problem is the matrix equation

$$\frac{d}{dt}\Psi=\begin{pmatrix} -i\zeta & u & v \\ -u^* & i\zeta & 0 \\ -v^* & 0 & i\zeta \end{pmatrix}\Psi, \tag{3}$$

where Ψ is a three-element column vector, whose elements are complex functions of z and t, and $\zeta\equiv\xi+i\eta$ is the complex eigenvalue. The two polarization components, u and v, act as potentials in this scattering problem. The main ingredient of the IST is to find the functions u and v for the given scattering data, i.e., a set of the eigenvalues ζ, which is the discrete component of the scattering data, and coefficients that determine the asymptotic form of the functions

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$\Psi(t)$ as $t \rightarrow \pm\infty$. The latter evaluated at the eigenvalues provide a set of normalization coefficients which for real values of ζ constitute the continuous component of the scattering data.

In general, the inverse scattering problem cannot be solved in a closed form, but it is explicitly solvable for the solitons. The solitons can be obtained from a linear system of algebraic equations. For an N-soliton solution, the scattering data consist of two parts: N eigenvalues $\zeta_1, \ldots, \zeta_n$, and $2N$ residues of the scattering matrix (see [13] for details), $C_{11}, \ldots, C_{1N}$ and $C_{21}, \ldots, C_{2N}$. As u and v evolve with z according to Eqs. (1) and (2), the eigenvalues $\zeta_1, \ldots, \zeta_N$ remain constant, while the residues have the simple z dependence:

$$C_{1n} = c_{1n}\exp(2i\zeta_n^2 z), \quad n = 1 \ldots N, \tag{4}$$

$$C_{2n} = c_{2n}\exp(2i\zeta_n^2 z), \quad n = 1 \ldots N, \tag{5}$$

where c_{1n} and c_{2n} are complex constants. Since these constants and the eigenvalues are arbitrary, the N-soliton solution to the Manakov system has $6N$ free parameters, which can be compared with $4N$ parameters for the single-component NLS soliton. The complexity of the solution is therefore rapidly increasing with N. However, some free parameters are actually trivial and can be eliminated by means of symmetries of the governing equations. These include arbitrary displacements in t and z, the scaling transformation $z \rightarrow \mu^2 z, t \rightarrow \mu t, u \rightarrow u/\mu$, and the Galilean boost $z \rightarrow z, t \rightarrow t + \nu z, u \rightarrow u\exp(it\nu + i\nu^2 z/2)$ with arbitrary parameters μ and ν. These symmetries are common to both the NLS and the Manakov model. In addition, the Manakov model allows a rotation of the polarization, i.e., $(u,v) \rightarrow (u\cos\theta + v\sin\theta, -u\sin\theta + v\cos\theta)$, with an arbitrary angle θ, which can be used to eliminate yet another free parameter.

The IST for the Manakov system is known in two essentially different forms, based, respectively, on the Marchenko integral equation and on the Hilbert transformation. Both techniques involve rather tedious calculations, which we omit in favor of a discussion of the soliton solutions. A more detailed account of the Hilbert transformation method can be found in Ref. [13]. An alternative approach based on Hirota's method was recently proposed [11], although without the closed-form expressions for the two-soliton solution that we will present here. A deficiency of that method is that it does not provide any insight into how the scattering data affect the solution.

In this paper we will present the explicit form of the two-soliton solution, and more importantly, discuss how it is related to the scattering data. This will facilitate the understanding of the complicated solution a great deal. Along the way we manage to unify some previously published exact solutions [14–16], and show how they can be obtained as a limit of the two-soliton solution. The solutions published in [14–16] have it in common that they are stationary, i.e., the pulse shapes do not vary with propagation, hence they describe only the particular kind of the interaction dynamics when the attractive and repulsive forces between two pulses exactly balance. We will present the more general interaction scenario when the pulses form an oscillating breather state. This enables us to analytically explain the numerical findings in, e.g., [9], and to present an explicit expression for the pulse separation as a function of z. The latter is an important result which is likely to be of great use in the theory of soliton communication systems, since it analytically expresses how the interaction strength between neighboring pulses depends on the pulse separation and relative polarization. Finally, we will consider the merging-eigenvalue solutions to the Manakov system, which form a particular kind of interaction with a logarithmic, rather than linear or periodic, pulse divergence.

II. THE SOLITON SOLUTIONS

A. One eigenvalue—a single soliton

In the case of one discrete eigenvalue, the $N=1$ soliton can be written in a convenient vector form as

$$(u^*, v^*) = i\frac{\mathbf{c}}{|\mathbf{c}|}2\eta\frac{\exp[2i(\xi^2 - \eta^2)z + 2it\xi]}{\cosh[2\eta(t + t_0 + 2\xi z)]}, \tag{6}$$

where the initial position of the soliton is given by $t_0 = \ln(2\eta/|\mathbf{c}|)/2\eta$. The vector $\mathbf{c} \equiv (c_{11}, c_{21})$ has arbitrary complex components, which together with the complex eigenvalue $\zeta \equiv \xi + i\eta$ give us six parameters of the one-soliton solution. Note that each of these six parameters is explicitly related to a particular symmetry of the underlying equations: the phases of c_{11} and c_{21} determine arbitrary phase constants of the u and v fields, $|\mathbf{c}|$ determines the initial position t_0, the parameter $|c_{11}|/|c_{21}|$ determines the arbitrary polarization angle of the soliton, and, finally, the imaginary and real parts of the eigenvalue correspond to the above-mentioned Galilean and scaling invariances.

B. Two eigenvalues—the two-soliton solution

1. The general solution

In accordance with what was said above, the most general form of the Manakov two-soliton solution involves twelve parameters, so it rather quickly becomes quite complex. Therefore, we will first adopt some simplifying assumptions that will allow us to obtain useful but yet nontrivial results. First of all, soliton-soliton collisions have been well studied, so we are not interested in solitons that move relative to each other. Thus we will assume that the eigenvalues are purely imaginary and will consider comoving soliton states (breatherlike solutions) only. The general two-soliton solution to the Manakov system satisfying this limitation can be written as

$$(u^*, v^*) = 4\Delta^{-1}\sum_{n=1,2} Q_{mn}, \tag{7}$$

where $m=1$ and 2 correspond, respectively, to u and v, and

$$Q_{mn}=\eta_n(\eta_1+\eta_2)^4 C_{mn}\exp(2(\eta_n t+2\eta_{3-n})t)+(-1)^n\eta_{3-n}(\eta_1-\eta_2)(\eta_1+\eta_2)^3(|C_{1n}|^2+|C_{2n}|^2)C_{m,3-n}\exp(2\eta_{3-n}t)-2(-1)^n\times\eta_1\eta_2(\eta_1-\eta_2)(\eta_1+\eta_2)^2 C_{mn}\times(C^*_{1n}C_{1,3-n}+C^*_{2n}C_{2,3-n})\exp(2\eta_{3-n}t), \quad (8)$$

$$\Delta=(\eta_1+\eta_2)^4\exp(4(\eta_1+\eta_2)t)+4\eta_1\eta_2(\eta_1+\eta_2)^2\times(C_{11}C^*_{12}+C^*_{11}C_{12}+C_{21}C^*_{22}+C^*_{21}C_{22})\times\exp(2(\eta_1+\eta_2)t)+(\eta_1+\eta_2)^4[(|C_{11}|^2+|C_{21}|^2)\times\exp(4\eta_2 t)+(|C_{12}|^2+|C_{22}|^2)\exp(4\eta_1 t)]+4\eta_1\eta_2(\eta_1-\eta_2)^2|C_{11}C_{22}-C_{12}C_{21}|^2+(\eta_1-\eta_2)^4(|C_{11}|^2+|C_{21}|^2)(|C_{12}|^2+|C_{22}|^2). \quad (9)$$

Here, the quantities C_{mn} are the same as in Eqs. (4) and (5).

The solution (7) contains ten free parameters, as it assumes $\xi_1=\xi_2=0$. However, according to what was said above, four of these parameters can be eliminated by obvious symmetry transformations. Further, the sum of the eigenvalues gives the soliton's energy according to $\int_{-\infty}^{+\infty}(|u|^2+|v|^2)dt=4(\eta_1+\eta_2)$, which can be scaled out. Hence, there remain only five truly nontrivial parameters.

The solution (7) is still too complex for straightforward applications. Let us now consider some special cases, in order to obtain tractable but yet nontrivial explicit solutions, which will be of interest for physical applications.

2. Two nonzero residues

Setting two of the four residues c_{mn} equal to zero, one retrieves previously known elementary solutions to the Manakov system. We will briefly consider them here for the sake of completeness. We can distinguish three separate cases. First, if $c_{12}=c_{22}=0$, we obtain once again the one-soliton solution presented above. Second, if $c_{21}=c_{22}=0$, we conclude that $v=0$. Hence we obtain the ordinary two-soliton solution to the NLS equation.

The third case is more interesting. It corresponds to $c_{12}=c_{21}=0$, so that the resulting two-soliton solution has six free parameters. After dropping the arbitrary phase constants in u and v, and making use of the scaling transformation to scale the eigenvalues according to the normalization condition $\eta_{1,2}=\frac{1}{2}\pm a$, the resulting two-soliton solution has three free parameters with a being a fourth one. This two-soliton solution can be cast into the form

$$u=\frac{4\eta_1[\exp(\theta_2)+2a\exp(-\theta_2)]\exp(2i\eta_1^2 z)}{2\cosh(\theta_1-\theta_2)+\exp(\theta_1+\theta_2)+4a^2\exp(-\theta_1-\theta_2)}, \quad (10)$$

$$v=\frac{4\eta_2[\exp(\theta_1)-2a\exp(-\theta_1)]\exp(2i\eta_2^2 z)}{2\cosh(\theta_1-\theta_2)+\exp(\theta_1+\theta_2)+4a^2\exp(-\theta_1-\theta_2)}, \quad (11)$$

where we have introduced the notation $\theta_1=2\eta_1 t+t_1$, $\theta_2=2\eta_2 t+t_2$, $t_1=\ln(2\eta_1/|c_{11}|), t_2=\ln(2\eta_2/|c_{22}|)$. This solution has been discovered previously by Tratnik and Sipe [14], and in a simplified form ($t_1=t_2=0$) by Christodoulides and Joseph [15]. Menyuk [6] has presented a more general solution than Eqs. (10) and (11) by allowing complex eigenvalues, i.e., $\xi_{1,2}=\pm\delta$.

A characteristic feature of the solution (10) and (11) is that it is stationary, i.e., the corresponding intensity profiles do not change with z (the more general solutions corresponding to the complex eigenvalues [6] are nonstationary; they describe a collision between two solitons). These profiles include two-humped structures [14,15]. Such a solution in the form of two pulses which propagate as a stationary co-moving state, with no apparent interaction, does not exist in the scalar NLS equation, and is therefore a peculiarity of the Manakov model. A particularly interesting feature of this two-soliton solution seems to have been missed in previous works. Starting with Eqs. (10) and (11), one then sets $t_1=t_2=\frac{1}{2}\ln(2a)$. Then one finds that u is symmetric and v is antisymmetric with respect to $t=0$:

$$(u,v)=\sqrt{2a}\frac{(4\eta_1\cosh(2\eta_2 t)\exp(i2\eta_1^2 z),4\eta_2\sinh(2\eta_1 t)\exp(i2\eta_2^2 z))}{2a\ \cosh(2t)+\cosh(4at)}. \quad (12)$$

In the limit $a\ll 1$, this solution can be written in the approximate simplified form,

$$(u,v)\approx[\text{sech}(t-t_0)\pm\text{sech}(t+t_0)]e^{2i\eta_{1,2}^2 z}/\sqrt{2}, \quad (13)$$

where $\cosh(2t_0)\equiv 1/(2a)$. In Fig. 1, this solution is displayed for different small values of a. This shows that it is indeed possible to obtain two effectively noninteracting stationary pulses of the same shape, separated by an arbitrary distance. This requires, however, that their initial shapes and relative polarization be carefully adjusted. Note also that the polarization of each soliton is slowly rotating with the period $\pi/4a$. Thus, the interaction between solitons in the Manakov model gives rise not only to motion of their centers but also to rotation of their polarizations. This fact has been observed also in more general systems than the Manakov model [17].

The fact that the interaction between the solitons could be canceled was originally observed numerically [16], but Eq. (12) is the first analytical explanation of this phenomenon. In practical communication systems, this possibility to effectively cancel the interaction between adjacent solitons by using proper polarization looks quite promising. However, due to difficulties in launching the exact pulse shapes and polarization states a complete cancellation may not be possible to achieve in practical communication systems. Nevertheless, already without knowing the exact solution (12) orthogonal polarization of adjacent solitons has been demonstrated to increase the bitrate with a factor of nearly 2 [8]. A physical reason for the effective absence of interaction between the two pulses in Eq. (12) is that we have a balance between repulsive and attractive interaction forces. The repulsive force originates from the v components of the pulses which are π out of phase, and the attractive force is produced by the u components which are in phase. Notice that effects generated by competing interactions between two different components of solitons to a Ginsburg-Landau equation were recently considered in Ref. [18].

3. Four nonzero residues

This is the most general case, which corresponds to the solution (7) (recall that we are dealing only the purely imaginary eigenvalues). To cast the solution into a more tractable form, we will now make an additional simplification, assuming all four residues c_{mn} to be in-phase: $c_{mn} \equiv -ik_{mn}$ with real k_{mn}. This choice removes only two free parameters, as the other two are the arbitrary phase constants of the u and v fields. Eventually, this leaves us with a soliton solution having six free parameters. It is also convenient to introduce the vector notation for the residues as follows: $\mathbf{k}_1 = (k_{11}, k_{21})$, $\mathbf{k}_2 = (k_{12}, k_{22})$, so that the two-soliton solution can be written as

$$(u,v) = \frac{2[e^{i2\eta_1^2 z}(\mathbf{t}_1 e^{-2t\eta_2} + \mathbf{k}_1 e^{2t\eta_2}) + e^{i2\eta_2^2 z}(\mathbf{t}_2 e^{-2t\eta_1} + \mathbf{k}_2 e^{2t\eta_1})]}{e^{2t(\eta_1+\eta_2)} + n_1 e^{-2t(\eta_1+\eta_2)} + n_2 e^{2t(\eta_2-\eta_1)} + n_3 e^{2t(\eta_1-\eta_2)} + n_4 \cos(2z(\eta_1^2-\eta_2^2))}, \tag{14}$$

where

$$\mathbf{t}_1 = [\mathbf{k}_1 |\mathbf{k}_2|^2 (\eta_1+\eta_2) - \mathbf{k}_2 (\mathbf{k}_1 \cdot \mathbf{k}_2) 2\eta_2] \frac{\eta_1 - \eta_2}{[2\eta_2(\eta_1+\eta_2)]^2}, \tag{15}$$

$$\mathbf{t}_2 = [\mathbf{k}_2 |\mathbf{k}_1|^2 (\eta_1+\eta_2) - \mathbf{k}_1 (\mathbf{k}_1 \cdot \mathbf{k}_2) 2\eta_1] \frac{\eta_2 - \eta_1}{[2\eta_1(\eta_1+\eta_2)]^2}, \tag{16}$$

$$n_1 = \left(\frac{\eta_1 - \eta_2}{4\eta_1\eta_2(\eta_1+\eta_2)^2}\right)^2 [|\mathbf{k}_1|^2 |\mathbf{k}_2|^2 (\eta_1+\eta_2)^2 - 4(\mathbf{k}_1 \cdot \mathbf{k}_2)^2 \eta_1 \eta_2], \tag{17}$$

$$n_2 = \frac{|\mathbf{k}_1|^2}{4\eta_1^2}, \tag{18}$$

$$n_3 = \frac{|\mathbf{k}_2|^2}{4\eta_2^2}, \tag{19}$$

$$n_4 = 2\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{(\eta_1+\eta_2)^2}. \tag{20}$$

A rich variety of solitons can be analyzed using this relatively simple explicit solution with its six free parameters. We defer this to a later work, and focus here on the interaction between pulses with the same shape. Then we can further simplify the soliton by choosing a reference plane of polarization and a temporal constant. We do this so that the u polarization becomes symmetric and the v polarization becomes antisymmetric with respect to $t=0$. This choice renders the denominator of the solution (14) an even function of t, i.e., $n_1 = 1$ and $n_2 = n_3$. The numerators of the solution must then be, respectively, even and odd functions of t for the u and v components. It is possible to show that these restrictions reduce the number of the free parameters to three. Finally, this number is reduced to two by normalizing the eigenvalues as it was done above, i.e., $\eta_1 = \frac{1}{2} + a$ and $\eta_2 = \frac{1}{2} - a$. The resulting soliton solution becomes

$$u(t,z) = 2\frac{k_{11}\cosh(2t\eta_2)e^{2i\eta_1^2 z} + k_{12}\cosh(2t\eta_1)e^{2i\eta_2^2 z}}{\cosh(2t) + n_2\cosh(4ta) + \frac{1}{2}n_4\cos(4za)}, \tag{21}$$

$$v(t,z) = 2\frac{k_{21}\sinh(2t\eta_2)e^{2i\eta_1^2 z} + k_{22}\sinh(2t\eta_1)e^{2i\eta_2^2 z}}{\cosh(2t) + n_2\cosh(4ta) + \frac{1}{2}n_4\cos(4za)}. \tag{22}$$

Writing the residue vectors as $\mathbf{k}_{1,2} \equiv |\mathbf{k}_{1,2}|(\cos(\phi_{1,2}), \sin(\phi_{1,2}))$ enables us to express the five parameters, $\mathbf{k}_1, \mathbf{k}_2$, and a in terms of the two angles ϕ_1 and ϕ_2 as follows:

$$2a = \frac{\tan(\phi_2 - \phi_1)}{\tan(\phi_2 + \phi_1)}, \tag{23}$$

$$|\mathbf{k}_{1,2}|^2 = \frac{\sin^2(2\phi_{2,1})}{2\sin^2(\phi_2-\phi_1)\cos(\phi_2-\phi_1)\cos(\phi_2+\phi_1)}. \tag{24}$$

This form of the Manakov two-soliton solution is appropriate for the study of interactions between arbitrarily polarized pulses, and it is possible to translate the two free parameters into a separation and a relative polarization angle between the solitons. In particular, by noting that two sech pulses can be described as $\mathrm{sech}(t+\tau) + (t-\tau) = 4\cosh(t)\cosh(\tau)/[\cosh(2t)+\cosh(2\tau)]$ we can identify the denominators of the solution (21) and (22) with $\cosh(2t)+\cosh(2\tau)$. This is valid for $a \ll 1$, when the pulse separation is large and we can then write the separation 2τ as a function of z as

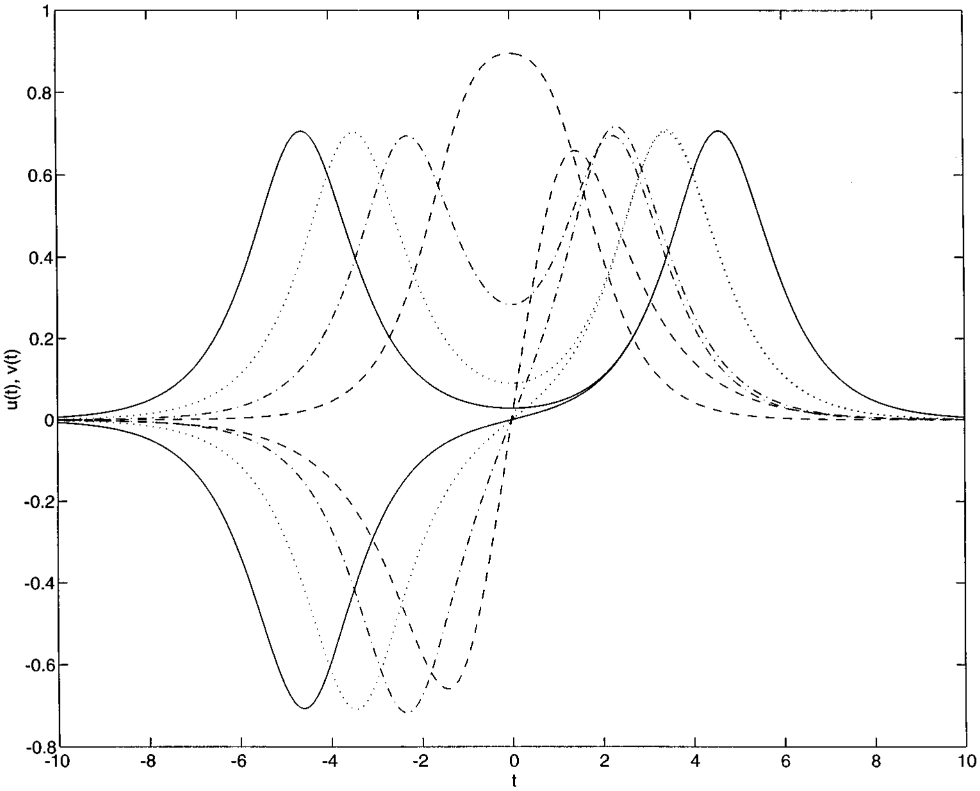

FIG. 1. The stationary solution Eq. (12) for $a=0.1$ (dashed), 0.01 (dash-dotted), 0.001 (dotted), and 0.0001 (solid). The $u(v)$ field is (anti)symmetric around $t=0$.

$$2\tau \approx \ln[2n_2+n_4\cos(4az)] = \ln\left[\frac{1}{a}\,\frac{\sin(\phi_2+\phi_1)}{\sin(\phi_2-\phi_1)}[1+\cos(\phi_2-\phi_1)\cos(4az)]\right]. \tag{25}$$

It is straightforward to show that this expression reduces to the NLS result [12] $\tau=\ln|\cos(2az)/a|$ in the limit $\phi_{1,2}\rightarrow 0$. In fact, in this limit the u field (21) reduces to the NLS-two-soliton solution, whereas the v field (22) vanishes. Another interesting limit is when the coefficient in front of the cos term of Eq. (25) vanishes, which occurs when $\mathbf{k}_1$ and $\mathbf{k}_2$ are orthogonal. This is the stationary, effectively noninteracting solution (12) discussed previously, and it can be recovered from Eqs. (21)–(24) in the limit $\phi_2\rightarrow\pi/2$, $\phi_1\rightarrow 0$. Finally, one may also note that the simulated case in Ref. [9] (also discussed in Ref. [2]) corresponds to Eqs. (21)–(24) with $a\approx 0.01$ and $\phi_2\approx 1.0$.

To express the polarization state of the pulses in terms of the residue angles, however, is more involved, especially since the polarization state may vary across a single pulse. We defer that, together with a more complete analysis of the solution (21) and (22), to a later publication.

4. The two-soliton solution with merging eigenvalues

The physical relevance of all the particular solutions obtained above (as well as of those found in Refs. [14] and [15]) is limited by the well-known fact that all the two-soliton solutions to the scalar NLS equation and the Manakov system are, strictly speaking, unstable, as their ''binding energy'' is exactly equal to zero. Hence they can be destroyed by an arbitrary perturbation of the initial state [5]. Moreover, it was shown [17] that two-humped solitons are unstable in more general (nonintegrable) models as well. Nevertheless, this does not mean that the two-soliton solutions are of no physical interest, as this instability may be in many practically important cases a fairly weak effect, which does not prevent the occurrence of two-soliton states in an experiment. Indeed, originally developed soliton lasers produced two solitons [19], although lasers producing single solitons have recently been available, too [19,20].

Another physically meaningful solution can be obtained as a limiting form of the ones found above in the case of two eigenvalues merging and becoming degenerate. Indeed, in a real optical communication line, one usually tries to create an array of identical fundamental solitons. However, various small perturbations give rise to a ''jitter'' of the solitons [2], which will eventually lead to a small difference in the amplitudes and phases of neighboring solitons in the array. In the case when this difference is much smaller than the amplitudes of the solitons, their interaction is described by this limiting solution. Therefore, this seemingly degenerate solution has, as a matter of fact, a special physical relevance.

For the NLS equation, the limiting form of the two-soliton solution was obtained already in the original work of Zakharov and Shabat [5], and more recently a slight generalization with an additional free parameter was reported by Gagnon *et al.* [21]. It was found that, for this special solution, the separation between the pulses grows with z proportional to ln$z$, rather than varying periodically or linearly with z as is the case for the general two-soliton solution.

Taking this limit of the solution described by Eqs. (21)–(24), which corresponds to $a\rightarrow 0$ and $\phi_1,\phi_2\rightarrow\phi_0$, we find that u and v take the form

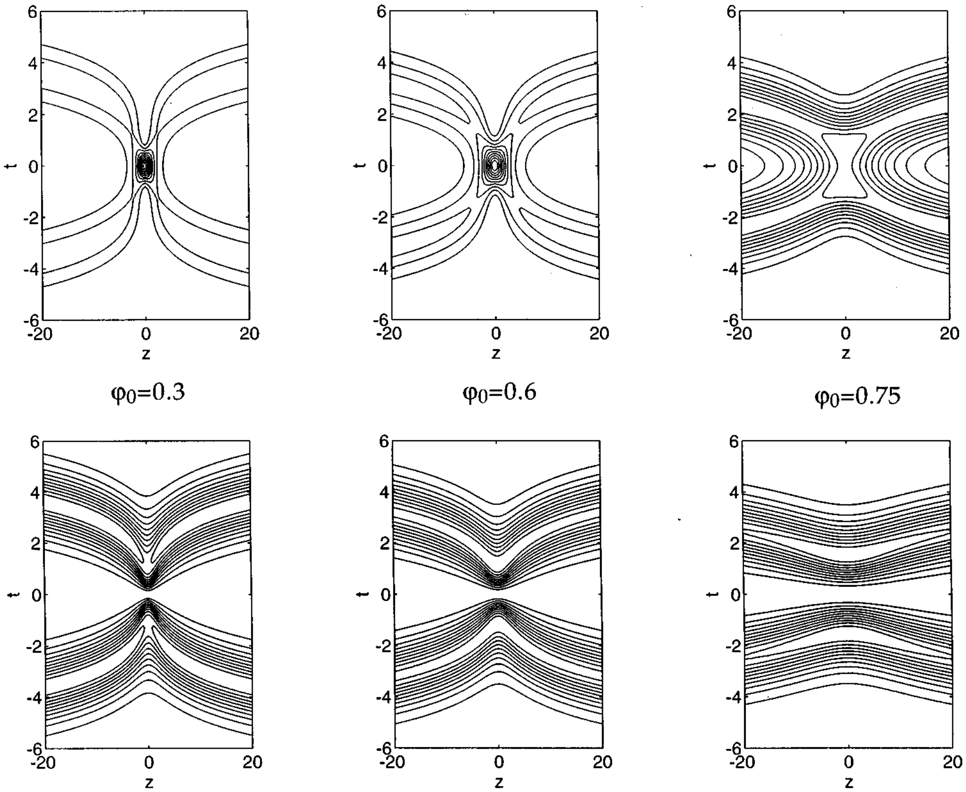

FIG. 2. The solution with merging eigenvalues, Eqs. (26) and (27), for $\phi_0=0.3$, 0.6, and 0.75. The upper row shows $|u(z,t)|^2$, and the lower row shows $|v(z,t)|^2$.

$$u(t,z)=4\frac{\cos(\phi_0)}{\sqrt{\cos(2\phi_0)}}\frac{\cos^2(\phi_0)\cosh t+\cos(2\phi_0)[iz\cosh t-t\sinh t]}{\cosh(2t)+\cos(2\phi_0)[2t^2+2z^2+1+\tan^2(2\phi_0)/2]}e^{iz/2}, \tag{26}$$

$$v(t,z)=4\frac{\sin(\phi_0)}{\sqrt{\cos(2\phi_0)}}\frac{-\sin^2(\phi_0)\sinh t+\cos(2\phi_0)[iz\sinh t-t\cosh t]}{\cosh(2t)+\cos(2\phi_0)[2t^2+2z^2+1+\tan^2(2\phi_0)/2]}e^{iz/2}, \tag{27}$$

where $0<\phi_0<\pi/4$ is the only remaining free parameter. In the limit $\phi_0\rightarrow 0$, we recover the known solution for the NLS equation [21]. One may note that the extra free parameter of the solution found by Gagnon *et al.* [21] destroys the symmetry of the solution with respect to the point $t=0$, and therefore it is not possible to retrieve it from Eqs. (21)–(24). However, using the more general expression (14)–(20), one can retrieve those asymmetric solutions as well. In Fig. 2, the solution (25) and (26) is shown for different values of the parameter ϕ_0.

III. CONCLUSION

In conclusion, we have derived the two-soliton solution to Manakov's equations in the case of purely imaginary eigenvalues. The two-soliton solutions have been studied in more detail in some physically relevant cases, including the case of the merging eigenvalues. The physical relevance of the solutions found has been discussed. The solutions are likely to be of value for further studies of soliton interaction in the optical fibers. It is also worth noting that the obtained solutions can give clues to the interaction dynamics between solitary waves in the frequently studied nonintegrable models that resemble the Manakov system, see, e.g., [2,6,9,17,19].

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